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Synopsis

On

A study on Power Graph of Finite Groups and related results

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Title: A study on Power Graph of Finite Groups and related results

Abstract:

In this research work, we would like to study the vertex and edge connectivity of power graph of finite groups, disconnecting sets in power graph, the complement of power graph, Also we would like to study different types of spectrum of power graph and calculate corresponding energies.

1 Introduction:

The research work on power graph of finite group have been proved by many scholars. The concept of directed power graph $\underset{\wp}{\rightarrow}$ (G) of a group G, introduced by Kelarev and Quinn [11]. Chakrabarty et al. [5] introduced the concept of an undirected power graph $\wp(G)$ of a group G.

Many researchers have contributed towards the understanding of power graphs of groups, especially after 2010. In 2018, Panda and Krishna [18] focused on the power graph of finite cyclic groups in general and obtained minimal separating sets of the power graph $\wp(\mathbb{Z}_n)$, which in turn gives the vertex connectivity of the power graph $\wp(\mathbb{Z}_n)$. For a given X $\subset$ $\mathbb{Z}_n$, X^c = $\mathbb{Z}_n$\ X and <X^c> is the induced subgraph of $\wp(\mathbb{Z}_n)$ induced by X^c.

In [19], Panda and Krishna determined minimum disconnecting sets of power graphs of finite cyclic groups, dihedral groups, dicyclic groups and abelian p-groups of finite order. Mehranian et al. [16] computed the spectrum of the power graph of cyclic groups, dihedral groups, elementary abelian groups of prime power order. Chattopadhyay [7] obtained the Laplacian spectrum of $\wp(\mathbb{Z}_n)$ and $\wp(D_{2n})$ for particular values of n. Panda [20] considered various aspects of Laplacian spectra of power graphs of finite cyclic groups, dicyclic groups and finite p-groups.

Definition 1.1:

The directed power graph $\underset{\wp}{\rightarrow}$(G) of a group G is a digraph with vertex set G and for any a, b ∈G, there is a directed edge from a to b in $\underset{\wp}{\rightarrow}$(G) if and only if a^k = b, where k ∈ $\mathbb{N}$.

Definition 1.2:

Given a group G, the power graph $\wp(G)$ of G is the simple undirected graph with vertex set G and two vertices a, b ∈G are adjacent in $\wp(G)$ if and only if b ≠ a and $b^k = a$ or a^k = b, k∈ $\mathbb{N}$.

Definition 1.3:

A set of vertices T of a graph Γ is said to be a separating set or cut-set, if its removal increases the number of connected components of Γ. T is called a minimal separating set or minimal cut-set if none of its non-empty proper subset is a separating set. If T is of least cardinality, then it is called a minimum separating set or minimum cut-set of Γ. The cardinality of a minimum separating set is called the vertex connectivity of Γ and it is denoted by k(Γ).

Definition 1.4:

For any finite simple undirected graph Γ, the Laplacian matrix L(Γ) is given by L(Γ) = D(Γ) – A(Γ) , where A(Γ) is the adjacency matrix of Γ and D(Γ) is the diagonal matrix of vertex degrees. Clearly L(Γ) is a real symmetric matrix and so all its eigenvalues are real.

2 Review of literature and development in the subject area:

The concept of directed power graph $\underset{\wp}{\rightarrow}$(G) of a group G, introduced by Kelarev and Quinn [11], is a digraph with vertex set G and for any a, b ∈G, there is a directed edge from a to b in $\underset{\wp}{\rightarrow}$(G) if and only if a^k = b, where k ∈ $\mathbb{N}$. For a semi-group, it was first considered in [12] and further studied in [13]. Chakrabarty et al. [5] introduced the

concept of an undirected power graph $\wp(G)$ of a group G, which was defined as follows: Given a group G, the power graph $\wp(G)$ of G is the simple undirected graph with vertex set G and two vertices a, b € G are adjacent in $\wp(G)$ if and only if b ≠ a and $b^k = a$ or $a^k = b$, k∈ $\mathbb{N}$. After that the undirected power graph became the main focus of study by several authors in [1, 3, 4, 9, 14, 15].

Chattopadhyay and Panigrahi [6, 7], determined the tight lower bound for the vertex connectivity of power graphs corresponding to cyclic groups $\mathbb{Z}_n$ and gave exact value of $k(\wp(\mathbb{Z}_n))$ when n is a power of some prime number. After that, many researchers extended these results and gave an upper bound of $k(\wp(\mathbb{Z}_n))$ for different values of n. Also, the vertex connectivity of the dihedral group D_{2n}, dicyclic group Q_{4n}, non-cyclic finite nilpotent group and non-cyclic abelian group of finite order were computed. In 2015, Chattopadhyay and Panigrahi [7], obtained another lower bound for the vertex connectivity of power graphs of certain finite cyclic groups. the authors of [19] examine the relationship between vertex connectivity and the minimum degree of power graphs of finite groups. They first explain some necessary conditions under which the vertex connectivity and the minimum degree of power graphs of finite groups coincide and computed the minimum degree when the equality holds for cyclic groups. Also, they gave a necessary and sufficient condition for $k(\wp(\mathbb{Z}_n)) = \delta(\wp(\mathbb{Z}_n))$.

Mehranian et al. [16] computed the spectrum of the power graph of cyclic groups, dihedral groups, elementary abelian groups of prime power order. The authors calculated the characteristic polynomial of $\wp(\mathbb{Z}_n)$ and $\wp_0(\mathbb{Z}_n)$. Chattopadhyay et al. [8] obtained both upper and lower bounds for the spectral radius of the power graph of $\mathbb{Z}_n$ and characterized the graphs for which these bounds are extremal. Further, they computed spectra of power graphs of the dihedral group D_{2n} and dicyclic group Q_{4n} partially and gave bounds for the spectral radii of these graphs.

In [17] give interesting facts about Laplacian eigenvalues of a graph. It is known that [10], the Laplacian eigenvalue with multiplicity 0 of a graph Γ is equal to the number of connected components of Γ. The Laplacian spectrum of a graph has a number of applications, including random walks, expansion properties, and statistical efficiency and optimality properties. In [2] for some of these. Chattopadhyay [7] obtained the Laplacian spectrum of $\wp(\mathbb{Z}_n)$ and $\wp(D_{2n})$ for particular values of n. In fact, the relationship between the spectrum of these two power graphs are discussed. Also, they gave sharp lower and upper bounds for algebraic connectivity of $\wp(\mathbb{Z}_n)$.

Panda [20] considered various aspects of Laplacian spectra of power graphs of finite cyclic groups, dicyclic groups, and finite p-groups. More specifically, Panda [20] determined completely the Laplacian spectral radius of power graphs of all of these groups apart from the algebraic connectivity and its multiplicity.

3 Aim and Objectives of Research:

- To study the concept of power graph of finite groups.
- To study the vertex and edge connectivity of power graph of finite groups.
- To study the disconnecting sets in power graph.
- To study the complement of power graph.
- To study the spectrum of power graph and Laplacian Spectrum of power graphs.
- To calculate different energies of power graph

4 Methodology to be adopted:

- Survey and collect relevant research papers, review articles, books, journals etc.
- Participate in conferences, workshops and seminars on relevant topic.
- Use computational software like Maple, Mathematica, Mathcad, for numerical calculations and graphical representations.
- Understand the results proved by the authors and solve the unknown problems listed by them

4 Importance of study:

The study of graphical representation of an algebraic structure, especially a semigroup or a group become an energizing research topic over the recent couple of decades, prompting many intriguing outcomes and questions. With the help of graph theoretical results, we can conclude the behavior of algebraic structures. Many interesting open problems in this area makes this study more interesting and important.

5. Plan of work:

Sr.No	Period	Plan of the work	Targets to be Achieved
1	First year	Study of the background required for the research topic.	Good command over the subject and clear the course work.
2	Second year	Collecting the papers and analyze them.	Try to get some new results based on the previous work.
3	Third year	Work on some problems to be included in the thesis.	Submit at least two research papers
4	Fourth year	Publishing some papers and submitting the thesis.	Some more papers and thesis submission.

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